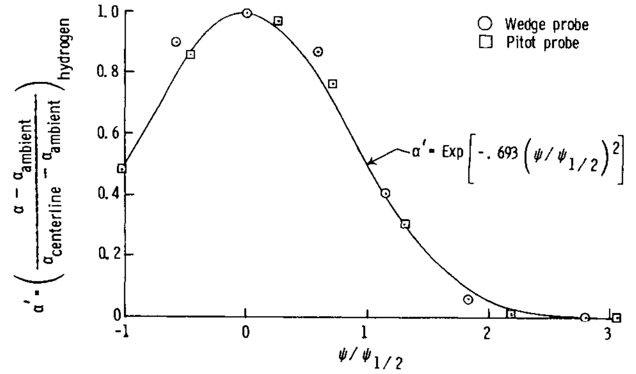

Fig. 4 Normalized hydrogen composition profiles.

analyzed by a gas chromatograph; since the collection cylinders were not heated, water in the samples condensed and results were obtained in the form of dry volume fractions of nitrogen, oxygen, and helium. Helium was used to trace the oxygen supplied to the burner.

Results and Discussion

Radial profiles of dry sample volume fractions as analyzed by the gas chromatograph are given for both probes in Fig. 2. Note that approximately the same radial distance was covered with each probe, but no attempt was made to match sample locations exactly. Results are given for both sides of the flow centerline to show symmetry, and extend through the mixing/reacting zone into the freestream. For all constituents the agreement between wedge and pitot results is quite good. Note also that the wedge sample at a radial distance of 0.72 cm contains both unreacted hydrogen and oxygen; this is probably due to unmixedness as described by several sources including Ref. 4.

To account for the water present in the samples, a data reduction computer program incorporating a mass balance and measured flow rates to the test gas supply was utilized. Plots of the resulting "wet" analysis which represents local stream composition are given in Fig. 3 in the form of calculated mass fractions of hydrogen, oxygen, nitrogen and water. Here, as in Fig. 2, the agreement between wedge probe and pitot probe data is good. The results appear reasonable in the sense that the region of depleted hydrogen and oxygen in Figs. 2 and 3 correspond as anticipated to a peak in the water profile, and the freestream values correspond closely to the bulk values computed from measured propellant flows to the burner.

A further validity check can be made by plotting the data in similarity-type form. According to Ref. 5, a concentration profile representable by a Gaussian distribution should be expected. Figure 4 shows such a plot, where the normalized concentration α' is the difference between local and freestream hydrogen concentrations divided by the difference in centerline to freestream values, and these concentrations have been converted to the unreacted state. The abscissa is the Von Mises coordinate ψ normalized by $\psi_{\frac{1}{2}}$, the value of ψ where α' is 0.5. The ψ values for the data points are computed by the technique described in Ref. 4; they are used in lieu of radial coordinates to eliminate the effect of heat release on the results. Both wedge and pitot data agree well with the Gaussian distribution, and are concluded to be reasonable.

Conclusions

Composition data obtained with the wedge and pitot probes agree well in regions with and without strong gradients. This implies that neither chemical nor boundary-layer effects induced by use of the wedge probe are different from those of the pitot probe. The wedge probe shows good survival qualities, produces results which appear to be valid, and allows total sampling time to be greatly reduced; it is therefore judged to be an accurate and useful data acquisition tool for small-scale, supersonic reacting flowfields.

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Basic Limitation in Microwave Measurement of Plasma Temperature

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THE electron temperature of shock-heated plasma can be determined, in principle, from microwave measurements. In all such determinations reported heretofore,[1–3] the electron temperature t_e is computed from three other measured or estimated plasma parameters—the radiation temperature t_r, and the power reflection and transmission coefficients, R and T, respectively—from the equation

$$t_r = (1-R-T)t_e = (1-R)(1-e^{-\alpha l})t_e \tag{1}$$

where α is the absorption per unit distance and l is the thickness of the plasma. [When T is negligible but a diffraction component is present, a modified form of Eq. (1) is used.[4]]

A major assumption implicit in Eq. (1) is a step rise in temperature at the plasma boundary. This assumption is usually not valid because the shock-heated plasma produced in the laboratory is enveloped in a thin thermal boundary layer that sustains the temperature difference between the plasma and the surrounding medium. It is the purpose of this Note to examine the effect of this boundary layer on microwave measurements of electron plasma temperature.

Power Flow

The available microwave noise power from a blackbody at a uniform temperature t can be approximated by ktB, where k is Boltzmann's constant and B is the bandwidth under consideration. (The error in this approximation is about $2.5f_0/t\,\%$,[5] where f_0 is the center rf of the system in gigahertz, and t is in Kelvin.) For a partially transparent (gray) body, the available microwave noise power may be approximated[6] by $ktB(1-e^{-\alpha l})$.

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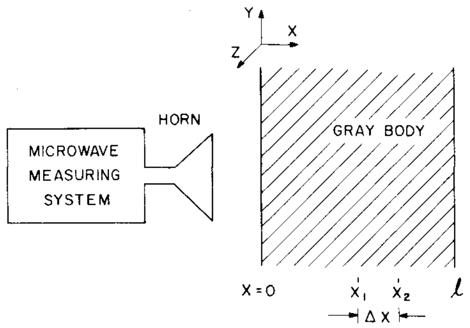

Fig. 1 Microwave system measuring the temperature of a gray body infinitely large in the y- and z-direction and l-cm thick in the x-direction.

Consider a gray body (Fig. 1) having a temperature t that is a well behaved function of x and independent of y and z. For simplicity, we assume that the rf impedance of the microwave system is matched to the effective impedance of the gray body and that the body is bounded on both sides by free space. Let the body be l centimeters thick in the x-direction and infinitely large in the y- and z-directions (i.e., much larger than the physical dimensions of the measuring antenna).

The noise power flowing to the left through a lamina of thickness Δx bounded by surfaces x_1 and x_2, neglecting internal reflections, is

$$P_1 = P_2 e^{-\alpha\Delta x} + ktB(1 - e^{-\alpha\Delta x}) \tag{2}$$

where P_1 and P_2 are the available powers at surfaces x_1 and x_2. Subtracting P_2 from both sides, expanding the exponential term in a power series, dividing both sides by Δx, and taking the limit as Δx approaches zero, yields

$$dP/dx = \alpha P - \alpha ktB \tag{3a}$$

Approximating P by $kt'B$ and dividing both sides of Eq. (3a) by kB yields

$$dt'/dx = \alpha(t' - t) \tag{3b}$$

(t' is the temperature corresponding to the available power P at any plane x, whereas t is the local temperature.) This first-order differential equation governs the noise power flow in the gray body.

Electron Temperature

Basically two microwave techniques have been used for determining the electron temperature of shock-heated plasma: In one,[3] the plasma is viewed in a waveguide probe positioned coaxially in the shock tube; in the other,[1,2,7] the plasma is viewed by microwave horns whose axes are perpendicular to the tube. In the waveguide-probe technique, the effect of the thermal boundary layer may be partially masked by the shock front which is generally not in thermal equilibrium.[8] We therefore restrict our attention to results obtained by microwave horns.

In this case, the measuring system is looking at a plasma slug having a temperature distribution, in the direction of the measured power flow, such as the one shown in Fig. 2. The temperature in Region II is assumed constant and equal to the value predicted by shock-wave theory.[9] The temperature profile in Regions I and III—representing the shock-tube sidewall boundary layers, and usually constituting only a relatively small fraction of the plasma thickness is assumed to have the shape predicted by Mirels[10,11] and Knöös.[12] We will treat the plasma as a gray body having at each point an absorption per unit distance α given by [13]

$$\alpha = (2)^{1/2}\frac{\omega}{c}\left\{\left[\left(1 - \frac{(\omega_p/\omega)^2}{1+(\nu/\omega)^2}\right)^2 + \left(\frac{\nu/\omega(\omega_p/\omega)^2}{1+(\nu/\omega)^2}\right)^2\right]^{1/2} - \left(1 - \frac{(\omega_p/\omega)^2}{1+(\nu/\omega)^2}\right)\right\}^{1/2} \tag{4}$$

where ω is the radian frequency under consideration; c is the speed of light; and ω_p is the local electron plasma frequency (in radians) given by $\omega_p^2 = n_e e^2/m\varepsilon_0$, with n_e the local plasma electron number density, e the electronic charge, m the electronic mass, and ε_0 the permittivity of free space; ν is the local collision frequency approximated by[14]

$$\nu = 3.62 \times 10^{-6}\frac{n_i}{t_e^{3/2}}\ln\left(1.23 \times 10^7\frac{t_e^{3/2}}{n_e^{1/2}}\right) + 2.6 \times 10^4\sigma^2 n_n t_e^{1/2} \tag{5}$$

where n_i, n_e, and n_n are the local ion, electron, and neutral densities (per cubic meter), respectively; and σ is the collision diameter (in meters) for electron-neutral collisions. The values of n_e, n_i, and n_n are taken to be those predicted by shock-wave theory for the local temperature at each point in the plasma cross section. Thus, by Eqs. (4) and (5), a given temperature profile establishes a unique α profile.

In the simple special case of relatively low shock levels—namely, at levels where the plasma frequency in the free-flow region (Region II) is below the rf of the microwave measuring system—the value of α in Region II (Fig. 2) is low, and significantly lower in most of the boundary layer (Regions I and III). Since the boundary layer is usually relatively thin, the effect of the boundary layer on the measured temperature is negligible and the assumption of a step rise in temperature at the plasma boundary is justified. Equation (1) is therefore a valid approximation for the electron plasma temperature of Region II, provided the diffraction is negligible and Region II is thick enough so that the contribution of the effective load temperature behind the plasma may be neglected. As the shock level increases, however, α increases, and the boundary layer plays an increasingly important role in determining what temperature the microwave system would measure.

For a given high shock level and a specified temperature profile in the boundary layer, we may predict the temperature that a microwave system would measure by integrating Eq. (3b) over the plasma cross section, using Eq. (4) for α. This can be readily accomplished by standard computer techniques.

As an example, consider argon, initially at 4 torr and room temperature, shock-heated at Mach 22 in a 5-cm-diam tube. Although shock-wave theory predicts[12] an equilibrium plasma temperature of 15,000 K, a double-horn X-band system such as the one used by Aro and Walsh[2] would measure a significantly lower temperature. To estimate this temperature, we need to know the temperature profile in the plasma cross section, along the direction of power flow into the measuring system. Knöös[12] predicts an exponential type of temperature profile across the boundary layer, assuming a laminar flow in the boundary; based on Mirels' theoretical work,[10,11] the thickness of the boundary layer at say 30 cm behind the shock front is about 0.3 mm. Integration of Eq. (3b) over the plasma cross section, at this point behind the shock front, shows that a microwave measuring system centered at say 9 GHz would yield a temperature of only 9000 K, or 40% lower than the theoretically predicted value. Moreover, since in our analysis we have neglected the multiple reflections within the plasma boundary layer, the difference may actually be significantly larger.

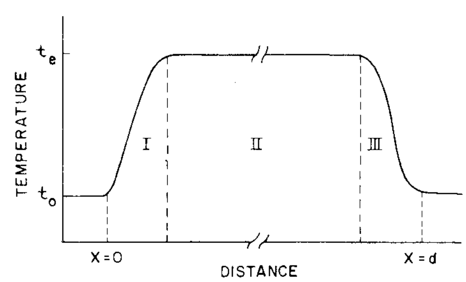

Fig. 2 Example of a transverse temperature profile of shock-heated plasma produced in the laboratory. Points $x = 0$ and $x = d$ represent the shock tube wall. t_0 is the initial temperature of the test gas; t_e is the equilibrium plasma temperature of the freestream plasma.

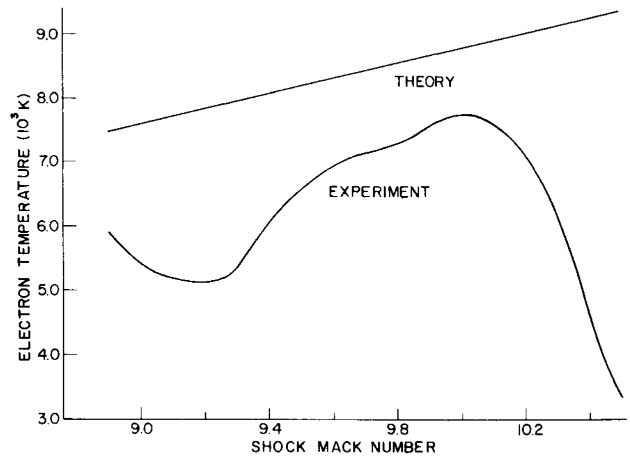

Fig. 3 Electron temperature of shock-heated argon initially at 4 torr and room temperature. The data for the experimental curve was obtained by Aro and Walsh[2] using a microwave-horn system whose *rf* was centered at 9.05 GHz; the curve incorporates the corrections of Ref. 4.

Conclusion

A basic limitation in microwave measurement of plasma temperature has been established: it has been shown that it is not possible to determine the electron temperature of shock-heated plasma by means of a microwave system operating at a frequency well below the plasma frequency because the plasma boundary layer exerts a prominent effect on the microwave measurements. The limitation applies, of course, to the determination of other plasma parameters as well, such as electron density and collision frequency. This limitation is responsible, in part, for the wide discrepancies between theory and experiment in the corrected[4] Aro-Walsh results[2] (Fig. 3) obtained by means of microwave horns for shock-heated argon at relatively high Mach numbers.

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Low Order Observer for a Linear Functional of the State Vector

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A PROCEDURE is described for the design of an observer of low order to provide a specified linear functional of the state vector of a linear system. The procedure yields information on the existence of an observer of given order, and on any constraints on the choice of observer poles.

There are many cases, e.g., in closed-loop pole assignment, where an observer is required to provide a specified linear functional of the state vector of a time-invariant linear system. A procedure for the design of such an observer, permitting the arbitrary choice of observer dynamics, was described in Ref. 2. However, as was shown by Fortmann and Williamson,[1] a reduction in observer order can be achieved by permitting the observer poles to be determined during the design process.

The method described by Fortmann and Williamson requires the reduction of the system to a number of single-output subsystems. In the present Note, a method is described, based on the procedure of Ref. 2, which is suitable for direct application to single-output or multioutput systems. The method permits the investigation of observers of increasing order, until an acceptable solution is found.

System Description

We consider a linear time-invariant system described by the equations

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{1}$$

where x, u, and y are vectors of state, input, and output, of dimension n, r, and m, respectively, and A, B, and C are constant matrices. The pair (A, C) is observable.

Problem Statement

The problem is to design an observer described by the equation

$$\dot{z} = Dz + Ky + Gu \tag{2}$$

where z is the q-dimensional observer state vector, and D, K, and G are constant matrices, such that, for a specified n-vector h^T, $(f^T y + g^T z)$ tends asymptotically to $h^T x$. The matrices D, K, and G and the row vectors f^T and g^T are to be found such that D has acceptable, but not necessarily arbitrary, eigenvalues, and the dimension q of the observer, is to be as small as possible.

Procedure

We postulate the existence of an observer of order q, to provide a linear functional specified by the vector h^T. Let the characteristic polynomial of D be

$$s^q + \beta_{q-1} s^{q-1} + \ldots + \beta_1 s + \beta_0 \tag{3}$$

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